Mathematical Representation of Creep for High-Temperature Performance of Nylon6.6 Tire Materials

L. NKIWANE, S. K. MUKHOPADHYAY

School of Textile Industries, University of Leeds, Leeds LS2 9JT, United Kingdom

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ABSTRACT: A creep model that describes the creep characteristics of nylon6.6 tire materials is proposed. An equation resulting from the model was used to fit the experimental curves by nonlinear regression. The results outline a good correlation between theoretical and experimental curves. © 1999 John Wiley & Sons, Inc. J Appl Polym Sci 72: 1505–1511, 1999

Key words: nylon6.6; tire cord; creep; temperature; strain; model; regression

INTRODUCTION

Creep is one of the time-dependent aspects of the mechanical properties of fibers which has considerable importance on short- and long-term product performance. Particularly, of the long-term physical properties critical to product acceptance in many engineering applications, creep is one of the most fundamental considerations. All fibrous materials creep to a greater or lesser extent and the general form of a creep curve is shown in Figure 1. The curve shows that when stress is applied to a material there is an instantaneous extension followed by rapid creep. This part is referred to as primary creep. After primary creep, there is a steady elongation referred to as secondary creep, then an accelerated creep leading to rupture known as tertiary creep. Some textile fibers fail before tertiary creep is reached.

Numerous empirical mathematical expressions have been proposed by researchers to describe experimentally observed creep data obtained at ambient conditions. In some cases, the derived relationships were only valid within the time interval conditions of the experiment. Models to help explain the creep behavior of materials have been developed by researchers such as Vangheluwe,[1] Bonner,[2] and Ward and Wilding.[3] Vangheluwe investigated the influence of the strain rate and yarn tex on tensile test results and proposed that the tensile curve can be described using a Maxwell element placed in parallel with a nonlinear spring. Bonner used the Eyring[4] approach to investigate the creep behavior of oriented polyethylene. This approach assumes that the deformation of a polymer is a thermally activated process involving the motion of segments of chain molecules over potential barriers. Ward and Wilding[3] used the same approach to investigate the creep behavior of ultrahigh-modulus polyethylene and found that there was a very good correlation between theoretical and experimental curves. This article proposes a theoretical model that describes the experimental creep characteristics of nylon6.6 tire cords observed over a range of temperatures in an isothermal mode of experiments.

A THEORETICAL MODEL

The proposed model is based on the Voigt element. A nonlinear spring is placed in series with the element. Similar models have been used by

Correspondence to: S. K. Mukhopadhyay.

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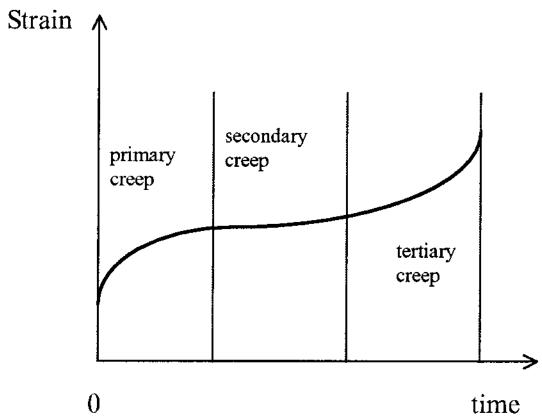

Figure 1 Schematic diagram of strain during creep for a typical viscoelastic material.

researchers such as Ward and Hadley,[5] Morton and Hearle,[6] and Young and Lovell.[7] The model is presented in Figure 2(a,b).

From Figure 2, the total strain e can be defined as

$$e = e_s + e_v \tag{1}$$

where e_s is the nonlinear spring strain, e_v is the strain for the Voigt element

$$e_v = e_1 = e_2 \tag{2}$$

e_1 is the strain in the linear spring, and e_2 is the strain in the dashpot.

The nonlinear spring can be defined using the equation

$$\sigma_s = ke_s^n \tag{3}$$

where k is the spring constant and the power n indicates a nonlinearity in the spring when $n \neq 1$.

Rearranging eq. (3) results in

$$e_s = (\sigma_s/k)^{1/n} \tag{4}$$

Since the nonlinear spring and the Voigt element are in series, the total strain of the system is

$$\sigma = \sigma_s = \sigma_v \tag{5}$$

where σ is the total stress for the Voigt element and

$$\sigma_v = \sigma_1 + \sigma_2 \tag{6}$$

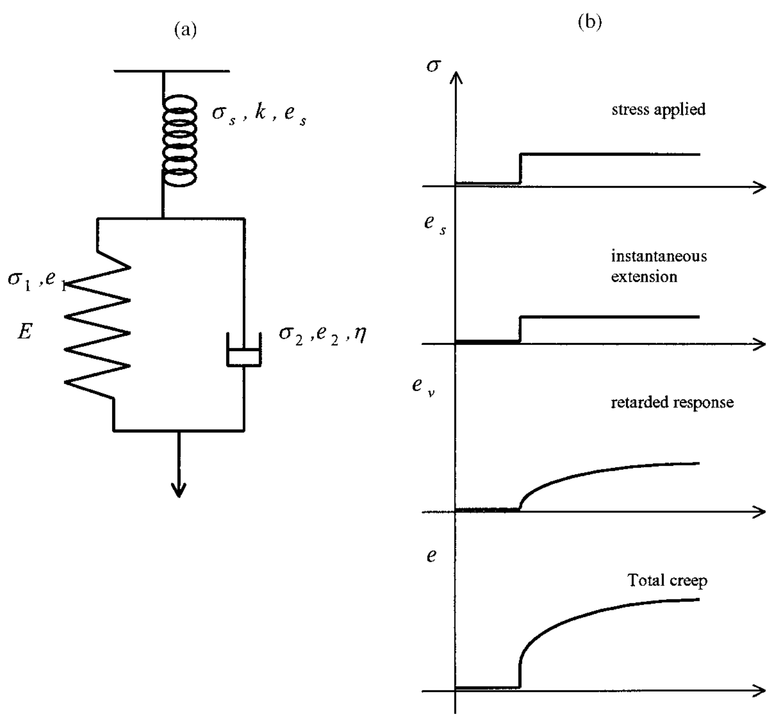

Figure 2 (a): A proposed model that describes the creep of nylon6.6 tire cords. (b) Creep behavior of the proposed model.

σ_1 and σ_2 being the stresses on the linear spring and dashpot, respectively. Equation (4) then becomes

$$e_s = (\sigma/k)^{1/n} \tag{7}$$

In the linear spring of the Voigt element,

$$\sigma_1 = Ee_1 \tag{8}$$

where E is the spring modulus, and in the dashpot,

$$\sigma_2 = \eta\ de_2/dt \tag{9}$$

where η is the viscosity. Since $e_1 = e_2 = e_v$, eqs. (8) and (9) can be written as

$$\sigma_1 = Ee_v \tag{10}$$

$$\sigma_2 = \eta\ de_v/dt \tag{11}$$

The dashpot describes the viscoelasticity of the material. At high constant temperatures (50°C and above), η will flow steadily as at low temperatures (e.g., room temperature around 20°C), but more rapidly. At conditions where the temperature was cycled between low and high (e.g., between 25 and 100°C), the material undergoes different creep phases such that η increases and decreases with temperature. The viscous flow is thus not steady and thereby the present theory will not apply for such nonisothermal experimental configurations.

Combining eqs. (6), (10), and (11), eq. (12) can be obtained:

$$\sigma_v = Ee_v + \eta\ de_v/dt \tag{12}$$

Rearranging eq. (12) results in

$$dev/dt = \sigma_0/\eta - Ee_v/\eta \tag{13}$$

In creep experiments,

$$\sigma = \sigma_0 = \text{constant} \tag{14}$$

and eq. (13) then becomes

$$de_v/dt = \sigma_0/\eta - Ee_v/\eta \tag{15}$$

Integrating eq. (15) results in

$$e_v = (\sigma_0/E)\{1 - \exp(-Et/\eta)\} \tag{16}$$

The above equations define the behavior of the model shown in Figure 2(a). The total strain for the model is given by eqs. (1), which yields

$$e = (\sigma_0/k)^{1/n} + (\sigma_0/E)\{1 - \exp(-Et/\eta)\} \tag{17}$$

The constant ratio η/E can be replaced by τ, the retardation time, so that the variation of the strain with time for the proposed model of a tire cord undergoing creep loading is given by

$$e = (\sigma_0/k)^{1/n} + (\sigma_0/E)\{1 - \exp(-t/\tau)\} \tag{18}$$

Perhaps it is appropriate to mention here that earlier researchers have found that most of the strain occurs within the retardation time.[8]

Equation (18) is fitted on experimental curves using nonlinear regression to enable the prediction of the creep of cords at any given time within the breaking limit. If the coefficient of determination R^2 for the fitted curve is high, there is a good dependence of creep on time, and the fit is good. R^2 is a measure of total variation of creep that is explained by eq. (18), a perfect fit being achieved when $R^2 = 1$. The values of R^2 were calculated using the Microsoft Excel computer program. This program enables the creation of graphs and also enables the determination of their equations and coefficients of determination R^2.

We have shown in eq. (18) that

$$e = (\sigma_0/k)^{1/n} + (\sigma_0/E)\{1 - e^{-t/\tau}\}$$

At instantaneous extension, $t = 0$, and the initial strain is therefore

$$e_0 = (\sigma_0/k)^{1/n} \tag{19}$$

and when $t = \infty$, the eventual strain

$$e_\infty = (\sigma_0/k)^{1/n} + (\sigma_0/E) \tag{20}$$

The strains in eqs. (19) and (20) are illustrated in Figure 3, which shows the creep curve for the untreated cord at 130°C (creep % = strain % − e_0%) and, therefore,

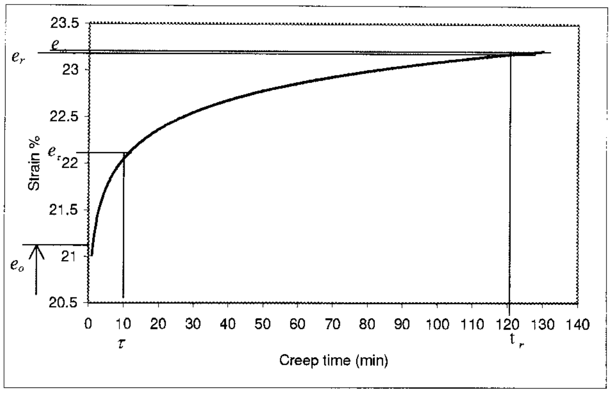

Figure 3 An illustration of initial strain (e_0) and strain at rupture (e_r) for untreated cord at 130°C. τ is the retardation time, and t_r, the breaking time.

$$e_\infty \approx e_r = (\sigma_0/k)^{1/n} + \sigma_0/E$$

$$= e_0 + \sigma_0/E \qquad (21)$$

where e_r is the strain at rupture. Hence, $\sigma_0/E = e_r - e_0$, resulting in

$$E = \sigma_0/(e_r - e_0) \qquad (22)$$

Further, when $t = \tau$,

$$e_\tau = (\sigma_0/k)^{1/n} + (\sigma_0/E)(1 - e^{-1})$$

$$= e_0 + (\sigma_0/E)0.63$$

Using eq. (21),

$$e_\tau = e_0 + 0.63(e_r - e_0) = 0.63\ e_r + 0.37\ e_0$$

Hence, if e_τ is known, from the experimental curves τ can therefore be found. When τ is known, and the value of E calculated using eq. (22), we have

$$\tau = \eta/E$$

resulting in

$$\eta = \tau E \qquad (23)$$

The values of E, e_τ, τ, and η are given in Table I. It can be observed that E, τ, and η increase with decreasing temperature. As the temperature increases, the molecular flow in the polymer becomes faster and therefore the amount of stress required to cause strain is reduced, the

Table I Calculated Values of Parameters as Outlined in Eqs. (17) and (18)

Material	$E \times 10^3$ (Nm^{-2})	$e_\tau \times 10^{-3}$ (m)	τ (min)	$\eta \times 10^5$ (Nm^{-2} min)
Yarn at 150°C	18.75	16.72	10	1.88
Yarn at 130°C	33.33	16.23	20	6.67
Yarn at 100°C	37.50	16.05	60	22.50
Yarn at 75°C	60.00	14.73	180	108.00
Yarn at 50°C	66.67	14.57	740	493.36
Untreated cord at 150°C	17.65	21.63	5	0.09
Untreated cord at 130°C	28.57	22.15	10	2.86
Untreated cord at 100°C	33.33	21.23	20	6.67
Untreated cord at 75°C	40.00	20.75	40	16.00
Untreated cord at 50°C	50.00	20.82	120	60.00
Untreated cord at room temperature (≈21°C)	66.67	20.47	2400	1600.08
Dipped cord at 150°C	15.38	21.35	5	0.08
Dipped cord at 130°C	20.69	21.63	5	1.04
Dipped cord at 100°C	35.29	21.13	10	3.53
Dipped cord at 75°C	46.15	20.72	30	13.85
Dipped cord at 50°C	54.55	20.59	60	32.73
Dipped cord at room temperature (≈21°C)	60.00	19.13	2040	1224.00

Table II Strains of Nylon6.6 Tire Cord Materials at 40% and 60% Stress Levels

Material	$e_{40} \times 10^{-3}$ (m)	$e_{60} \times 10^{-3}$ (m)	Multiplying Factor	n	k
Yarn at 150°C	23.5	29.4	1.25	1.81	19.39
Yarn at 130°C	24.6	30.2	1.23	1.96	24.40
Yarn at 100°C	24.5	29.9	1.22	2.03	28.23
Yarn at 75°C	23.1	28.9	1.21	2.11	25.80
Yarn at 50°C	23.0	28.6	1.23	2.35	60.92
Untreated cord at 150°C	30.8	38.9	1.23	1.67	9.36
Untreated cord at 130°C	32.9	41.6	1.26	1.73	9.08
Untreated cord at 100°C	32.9	40.1	1.22	2.05	16.09
Untreated cord at 75°C	33.0	39.6	1.20	2.20	21.16
Untreated cord at 50°C	32.5	38.8	1.23	2.30	24.31
Untreated cord at room temperature	33.4	39.7	1.19	2.35	26.66
Dipped cord at 150°C	30.2	38.5	1.23	1.67	9.78
Dipped cord at 130°C	31.8	39.5	1.24	1.87	12.40
Dipped cord at 100°C	30.9	37.9	1.23	1.99	17.62
Dipped cord at 75°C	32.7	39.7	1.21	2.09	11.89
Dipped cord at 50°C	31.8	38.5	1.21	2.12	18.39
Dipped cord at room temperature	30.5	36.7	1.20	2.19	24.73

[a] e_{40} and e_{60} are the instantaneous extension at the 40 and 60% stress levels, respectively.

time within which the strain takes place is also reduced, and the material becomes more viscous.

EXPERIMENTAL AND CURVE FITTING

Tests to determine the creep characteristics of nylon6.6 tire materials were carried out on yarn, an untreated cord, and a cord dipped in resorcinol formaldehyde latex resin. The cords were made by twisting together two yarns each of 140 tex. The tests were performed at temperatures of 150, 130, 100, 75, 50°C, and room temperature around 20°C, involving a newly built creep tester. All test samples had initial lengths of 200 mm. The experimental loads used were 40 and 60% of the breaking loads of the yarn, untreated and dipped cords as appropriate, measured at standard atmospheric conditions of 21°C temperature and 65% relative humidity. The strain profile of the yarn and

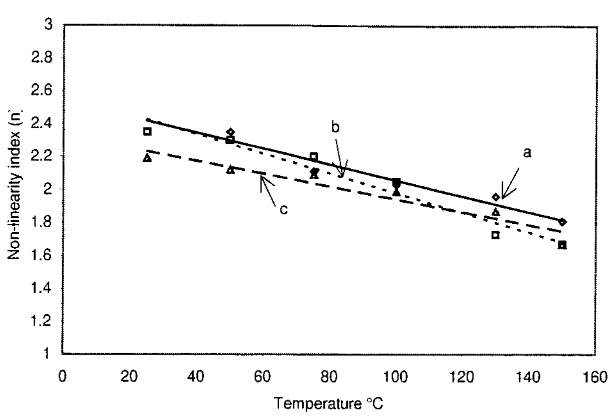

Figure 4 Effect of temperature on the behavior of nonlinear spring (a) yarn, (b) untreated cord, and (c) dipped cord.

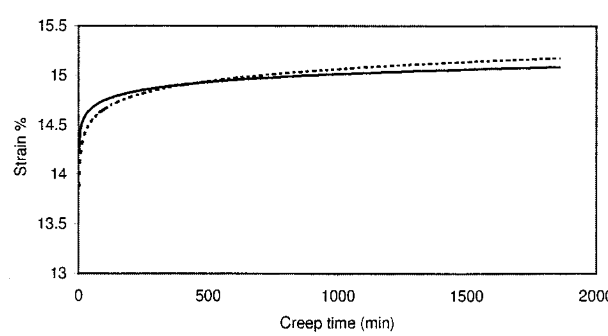

Figure 5 Experimental and theoretical creep curves for yarn at 75°C: (——) experimental; (---) theoretical.

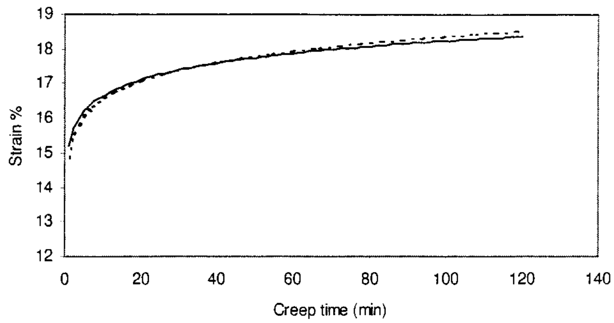

Figure 6 Experimental and theoretical creep curves for yarn at 150°C: (——) experimental; (---) theoretical.

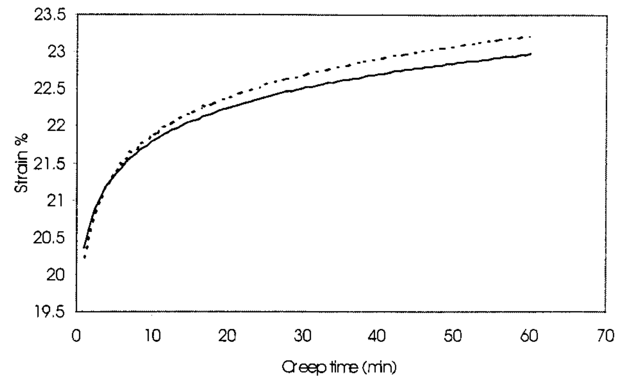

Figure 8 Experimental and theoretical creep curves for dipped cord at 130°C: (——) experimental; (---) theoretical.

untreated and dipped cords at 40% and 60% stress levels at different temperatures are given in Table II.

To find k and n, consider the following: If e is measured at levels of stress σ_0 = 40% and 60% of the ultimate breaking stress, the corresponding instantaneous strains are e_{40} and e_{60}. Thus,

$$\sigma_{40} = ke_{40}^n$$

$$\sigma_{60} = ke_{60}^n$$

Dividing

$$\sigma_{60}/\sigma_{40} = (e_{60}/e_{40})^n = 1.5$$

Hence,

$$n \log e_{60}/e_{40} = \log 1.5,$$

that is,

$$n = \log 1.5/\log e_{60} - \log e_{40}. \tag{24}$$

When n is known, we return to eq. (20) to find k (Table II):

$$k = \sigma_0/e_0^n \tag{25}$$

An examination of the values of the nonlinearity index n shown in Table II reveals that n decreases with increasing temperature and this trend is illustrated in Figure 4. There exist reasonably linear relationships between n and the experimental temperatures for the yarn, the untreated cord, and the dipped cord. Thus, it is not only the viscosity η that changes with temperature, but the behavior of the nonlinear spring changes as well.

The Voigt element placed in series with a nonlinear spring expresses the viscoelastic behavior of these tire cords. The strain resulting from applying stress σ_0 is given by eq. (18). Parameters in eq. (18) were calculated before this equation was fitted on the experimental curves (Figs. 5–8). There seems to be an agree-

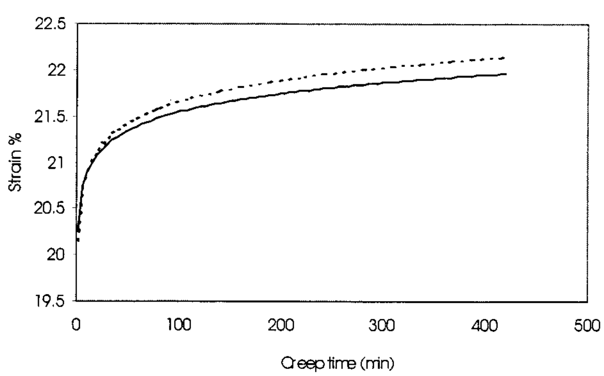

Figure 7 Experimental and theoretical creep curves for untreated cord at 100°C: (——) experimental; (---) theoretical.

Table III Values of Coefficient of Determination R^2

Material	R^2
Yarn at 75°C	0.944
Yarn at 150°C	0.936
Raw cord at 100°C	0.924
Dipped cord at 130°C	0.892

ment between the theoretical and experimental curves. The values of R^2 are around 0.9 (Table III), which shows a good fit of eq. (18) on the experimental curves and confirms the dependence of creep on time.

CONCLUSIONS

The model is suggested to represent the creep behavior of the nylon6.6 tire cords investigated. The proposed equation has a very good fit on the isothermal experimental creep curves. Further theoretical work is necessary to explain the nonisothermal creep behavior of nylon6.6 tire materials.

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